POSITION CONTROL OF PMSM IN SLIDING MODE

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Summary In the paper control of linear permanent magnet synchronous motor (PMSM) based on the principles of sliding mode control (SMC) with respect of vector control principles is carried out. The presented simulations comprise position quantization due to assumed experimental verification on the bench which consists of linear PMSM and incremental position sensor. Simulation results compare two methods for obtaining of position derivatives needed for SMC algorithm. The first method exploits a filtering observer and second one uses numerical derivations and first order filters.

1. INTRODUCTION

This paper presents SMC of a linear PMSM. Primary part of this linear PMSM is moving one and incremental position sensor yields quantized signal of position x_m.

Control of PMSM is based on principles of vector control, i.e. designed control technique divides PMSM into two channels. In d-channel the current component, i_d, is controlled at the zero value to control machine magnetic flux. In the q-channel position, x_m, is controlled on a demanded value with the specified dynamics to control machine force.

Both channels for control exploit principles of SMC, where only the plant rank, r, has to be known. As a result the control value derivatives up to the r-1 rank must create the feedback. Status of the system is then kept exactly in the r-1-dimensional switching boundary within the r-dimensional sub-state space.

Feedback system dynamics is determined only by boundary alone and doesn't depend on the system parameters and external disturbances and so a desirable robustness is ensured.

For more detailed description of control theory see [1], [2] and [3].

2. THEORY

A. Design of Sliding Mode Control Algorithm

Differential equations of linear PMSM extracted from [4] are as follows:

$$\frac{di_q}{dt} = \frac{1}{L_q}\left[u_q - R_s i_q - \frac{p}{r} v_m \left(\psi_{PM} + L_d i_d\right)\right], \quad (1)$$

$$\frac{di_d}{dt} = \frac{1}{L_d}\left[u_d - R_s i_d + \frac{p}{r} v_m L_q i_q\right], \quad (2)$$

$$F_e = \frac{3p}{2r}\left[\psi_{PM} i_q + (L_d - L_q) i_d i_q\right], \quad (3)$$

$$\frac{dv_m}{dt} = \frac{1}{m}\left(F_e - F_L\right), \quad (4)$$

$$\frac{dx_m}{dt} = v_m, \quad (5)$$

where:

$$r = p\tau_p / \pi, \quad (6)$$

A simple form of SMC is as follows:

$$u_{q\_dem} = u_{q\_\max} sign\left(S_\theta\right), \quad (7)$$

Demanded voltage, u_{q_dem}, is control variable, switching between maximal value u_{q_max} and $-u_{q\_max}$ which are imposed by the hardware.

Rank r=3 results from PMSM equations for q-channel. Then two derivatives of position, x_m, are needed for the feedback. Corresponding switching function, S_θ, is defined as:

$$S_\theta = x_{m\_dem} - x_m - w_1 \dot{x}_m - w_2 \ddot{x}_m, \quad (8)$$

The switching function switches whenever S_θ changes its sign. The equation of the switching boundary can be derived if S_θ is set to zero.

The control system is designed by the method of pole assignment, where the system has n poles of closed loop, i.e. n = r-1. The poles have equal real parts placed at, $\omega_0 = -1/T_c$, which ensure the fastest settling time of the step response without overshoot.

Ideal transfer function with respect of Dodds formula for specified settling time, T_s, is as:

$$\frac{x_m(s)}{x_{m\_dem}(s)} = \left[\frac{1}{1 + s\dfrac{T_s}{1{,}5(1+n)}}\right]^n \Bigg|^{n=2} = \frac{1}{s^2 \dfrac{4T_s^2}{81} + s\dfrac{4T_s}{9} + 1}, \quad (9)$$

The modification and the transformation into the time domain yields:

$$x_{m\_dem} - x_m - \frac{4T_s}{9}\dot{x}_m - \frac{4T_s^2}{81}\ddot{x}_m = 0, \quad (10)$$

Equating right hand sides of (10) and (8) yields control algorithm feedback gains:

$$w_1 = \frac{4T_s}{9}, \qquad w_2 = \frac{4T_s^2}{81}, \quad (11a,b)$$

Disadvantage of the control based on SMC principles is control chattering. This control chattering can be alleviated by inserting a pure integrator at the plant input. But this smoothing integrator together with the plant creates a 'new' plant to be of 1 rank greater than the original one. Therefore, it is necessary to repeat already introduced approach for r=4, from (7) to (11). In addition of this, if signum function is replaced by a proportional high gain, K_{SM}, then the control algorithm is as follows:

$$u_{q\_dem} = \int K_{SM}\left(x_{m\_dem} - x_m - \frac{T_s}{2}\dot{x}_m - \frac{T_s^2}{12}\ddot{x}_m - \frac{T_s^3}{216}\dddot{x}_m \right) dt \text{,(12)}$$

This means 1 rank higher derivation inputs to the control algorithm against the original plant. By integration of SMC algorithm (12) the highest derivative can be cancelled and the final position SMC algorithm is as follows:

$$u_{q\_dem} = K_{SM}\left[\int \left(x_{m\_dem} - x_m \right) dt - \frac{T_s}{2}x_m - \frac{T_s^2}{12}\dot{x}_m - \frac{T_s^3}{216}\ddot{x}_m \right] \text{,(13)}$$

This way, using the rearrangement of control system a smoothing integrator doesn't increase the number of derivatives in the feedback. The position control channel block diagram is shown in Fig. 1, which corresponds to (13) with mentioned rearrangement:

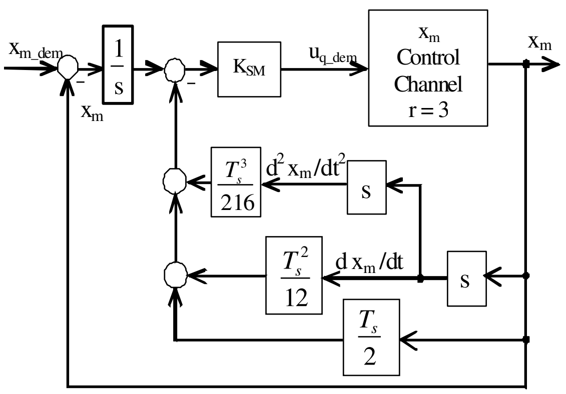


Fig. 1. SMC control loop for position x_m channel

Similar way the SMC algorithm is derived for current component, i_d control channel, which is as follows:

$$u_{d\_dem} = K_{SM\_i}\left[\int \left(i_{d\_dem} - i_d \right) dt - \frac{T_{si}}{3} i_d \right], \qquad (14)$$

The current component, i_d control channel block diagram corresponding with (14) is shown in Fig.2:

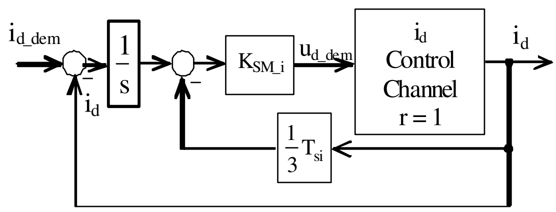


Fig. 2. SMC loop for current i_d channel

B. Position derivatives needed for SMC algorithm

One way to gain needed position derivatives, which are velocity, v_m, and acceleration, a_m, for SMC feedback position loop is using of observer. Detailed description of observer is in [3]. Therefore next section presents only short extract.

The observer is capable to produce filtered estimate of position, $\hat{x}_m$, estimate of velocity, $\hat{v}_m$, and acceleration, $\hat{a}_m$, including estimate the external load force, $\hat{F}_L$. State equations of the observer are as follows:

$$\frac{d\left(-\hat{F}_L\right)}{dt} = 0 + k_F e_x \qquad (15)$$

$$\frac{d\hat{v}_m}{dt} = \frac{1}{m}\left[\frac{3p}{2r}\psi_{PM} i_q + \left(-\hat{F}_L\right) \right] + k_v e_x \qquad (16)$$

$$\frac{d\hat{x}_m}{dt} = \hat{v}_m + k_x e_x \qquad (17)$$

The external force is assumed to be constant over a time interval that is short if compared with observer time constant, T_{so}. This assumption forms (15). The equations (16) and (17) are based on the PMSM model, (3), (4), (5). As current component, i_d, is controlled at the zero value then equation for electromagnetic force, F_e, can be reduced. Observer state equations include correction loops based on position error, $e_x = x_{m\_q} - \hat{x}_m$, which is defined as difference between the measured position and its estimate. Block diagram of the described observer is shown in Fig. 3.

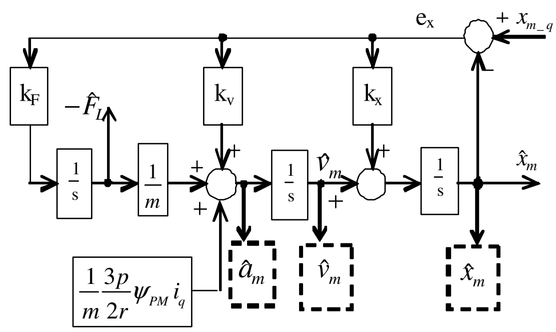


Fig. 3. Block diagram of observer

The correction loop gains k_x, k_v and k_F are designed by pole-placement. Equating the desired characteristic polynomial (*LHS* of (18)) and the characteristic polynomial of observer of Fig. 3 (*RHS)* of (18)):

$$s^3 + \frac{18}{T_{so}}s^2 + \frac{108}{T_{so}^2}s + \frac{216}{T_{so}^3} = s^3 + s^2 k_x + s k_v + \frac{k_F}{m} \qquad (18)$$

Desired characteristic polynomial was derived again with the aid of settling time formula (9) to yield a correction loop settling time, T_{so}.

Then correction loop gains are as follows:

$$k_x = \frac{18}{T_{so}} \;,\; k_v = \frac{108}{T_{so}^2} \;,\; k_F = \frac{216m}{T_{so}^3} \;. \quad (19)$$

Thank to inverse estimated load force, $-\hat{F}_L$, produced in the observer the characteristic polynomial (*RHS* of (18)) includes only plus signs and all the correction loop gains have positive values.

Block diagram of parallel SMC structure of SMPM with PWM modulation and with exploitation of filtering observer is shown in Fig. 4.

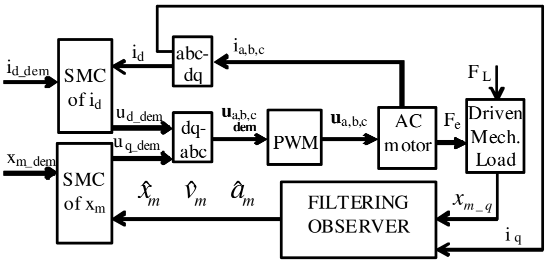


Fig. 4. Parallel SMC structure with filtering observer

The input to the filtering observer is position, x_{m_q}, which is quantized, as the output of the incremental position sensor. The quantization step is determined by the resolution of the used sensor.

Another way to gain the desired position feedback derivatives is numerical derivation. Before derivation the quantized position signal is filtered to gain smooth function of, x_{m_f}. Then the desired velocity is computed via numerical derivation and again filtered for elimination of noise. Filtered velocity, v_{m_f}, is again numerically derivates to gain desired acceleration, a_{m_f}, for feedback. All the filters exploited are of the first order.

Block diagram of parallel SMC structure with exploitation of numerical derivation to gain demanded feedbacks is shown in Fig. 5.

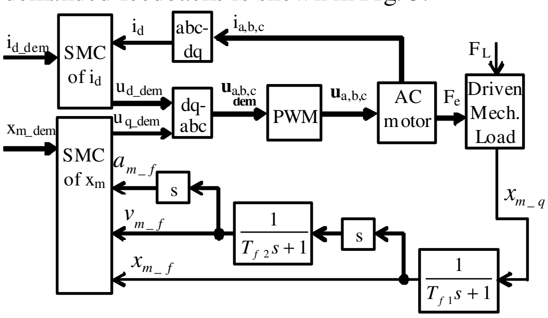


Fig. 5. Parallel SMC structure based on numerical derivation

3. EXPERIMENTAL RESULTS

The parameters of used linerar PMSM are: winding resistance, R_s=0.44 Ω, winding inductances, L_d=157 μH, L_q=141.3 μH, permanent magnet flux, Ψ_{PM}=0.066 Wb, pole pair number, p=3, pole pitch, τ_p=25mm, and weight of primary part, m=1.483kg.

All state variables introduced in theoretical section have zero initial states. The simulations presented further have following parameters: prescribed settling time of position, T_s=15 ms, position SMC algorithm gain, K_{SM}=7.10^6, prescribed settling time of d-current, T_{si}=5 ms, d-current SMC algorithm gain, K_{SM_i}=50. Computational step of control algorithm is 0,128 ms and SMPM model computational step is 10^{-6} s. Demanded position step, x_{m_dem}=5 mm, overall simulation time is 0,08 s and at the t=40 ms load force, F_L, is changed from zero value to 80 N to achieve approximately the nominal current 7A.

Simulation results for ideal SMC algorithm, where SMC algorithm and PMSM model without PWM modulation, without abc-dq transformations and position quantization, are shown in Fig. 6. Position derivatives as inputs of SMC algorithm are obtained directly from PMSM model. It can be seen from Fig. 6, that the demanded d,q-voltages and corresponding currents are smooth thanks to smoothing effect of the added integrator. The function of position x_m has also smooth behavior and as it’s evident from Fig. 6a the difference between real and ideal response x_{id} is very small therefore the demanded dynamics of position was also achieved.

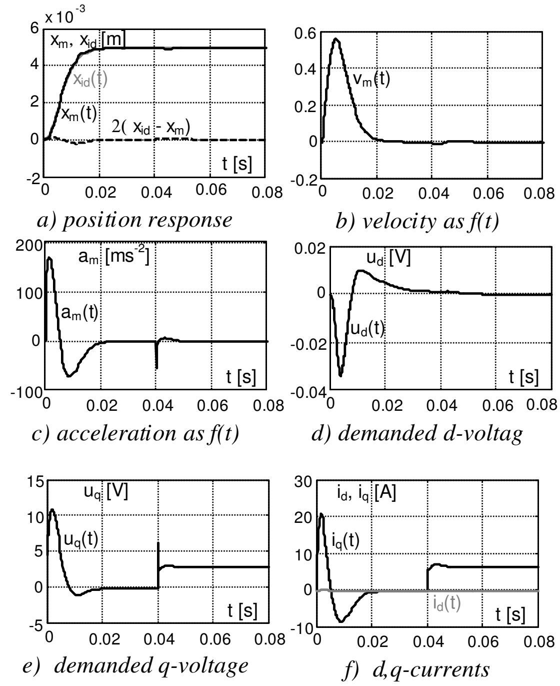
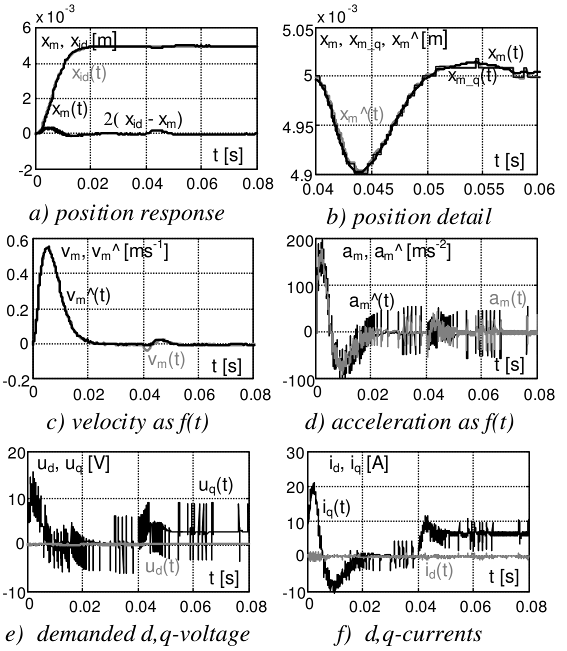


Fig. 6. Simulation results for ideal SMC

In next simulations PWM modulation, abc-dq transformations and position quantization is performed and DC-bus voltage U_{dc}=24 V and quantization is taken into account with computational step 10 μm.

Simulation results with filtering observer are shown in Fig. 7. Settling time of observer is T_{so}=5ms. Simulation results with numerical derivation for reconstruction of position derivatives are shown in Fig. 8, where time constants of the first order filters are T_{f1}= 0.001/3 and T_{f2}= 0.002/3.

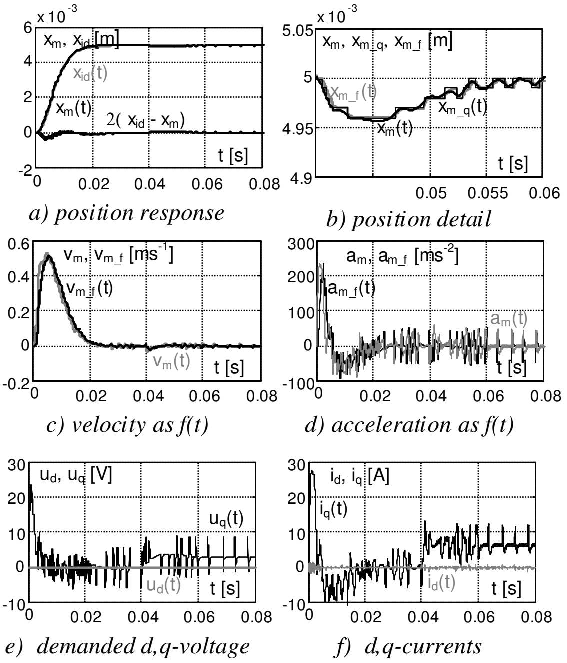


a) position response *b) position detail*

c) velocity as f(t) *d) acceleration as f(t)*

e) demanded d,q-voltage *f) d,q-currents*

Fig. 7. Simulation results for position SMC with exploitation of filtering observer



a) position response *b) position detail*

c) velocity as f(t) *d) acceleration as f(t)*

e) demanded d,q-voltage *f) d,q-currents*

Fig. 8. Simulation results for position SMC with numerical derivation for position derivatives

Fig. 7b shows quantization of PMSM model position as well as observed position, $\hat{x}_m$. Similar way in Fig. 8b are for short time interval shown quantized position, x_{m_q} and filtered position, x_{m_f}.

Although behaviors of velocity and acceleration in simulation with numerical derivation have a certain delays versus PMSM model, (*see Fig. 8c,d*) the position response shows only a small transient in position decrease due to loading by the same step of external force, F_L it's compared with the simulation with observer (*see Fig. 7b and Fig. 8b*). This position decrease in simulation with numerical derivation, Fig. 8a, is comparable with the decrease at ideal SMC shown in Fig. 6a. At simulation with observer the position transient is bigger due to certain error in observer estimated values, especially in velocity, (*see Fig. 7c*), during $\hat{F}_L$ transient. But interval with constant F_L hasn't any visible delay between model and its estimated behavior (*see Fig. 7c,d*).

It is valid for both simulations of Fig. 7 and Fig. 8 that chattering of u_q isn't caused by SMC but this chattering is a result of feedback derivatives. Also u_d is chattering, which is caused by chattering of i_d due to PWM modulation.

4. CONCLUSION

Based on presented simulation results for linear PMSM position control it can be concluded that chattering of u_d isn't significant and although i_d as an input of SMC algorithm would be filtered the chattering of i_d will be the same as a result of PWM.

A more significant is chattering of u_q resulting in chattering of position in steady-state. This can be eliminated by better filtering of position derivatives. But in simulation with the numerical derivation it's at the expense of control stability. For both simulations is also valid that transients in position error due to external force loading are more significant.

As conclusion is valid the smoother are derivatives as inputs of SMC algorithm the smoother is also i_q current and the only chattering presented is due to PWM modulation.

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